IDENTIFICATION OF THERMOPHYSICAL CHARACTERISTICS BY SOLVING INVERSE HEAT CONDUCTIVITY PROBLEMS

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A method is elucidated for the solution of internal inverse heat conduction problems that provides for the transformation of the initial mathematical model and the subsequent modeling of the thermal processes on analog apparatus.

The accuracy of temperature field computations depends greatly on the confidence in the data about the thermophysical properties of the bodies under investigation. Experimental methods of determining the thermophysical properties are tedious and assume the production of special experimental apparatus and the fabrication of specimens of completely definite geometry. In fact, every experiment is transformed into a unique investigation, which is not, by far, always easy to duplicate. In this connection, methods of solving inverse heat conductivity problems (IHCP) have recently started to be used extensively for the identification of the thermophysical characteristics of materials. A number of publications [1-4] are devoted to this question, in which different approaches are considered to the solution of interior IHCP (they are still called inverse, coefficient), and the regularity of the solutions obtained, the limits of applicability of the methods used, their realization by using different computer engineering facilities, etc., are studied. If we speak about the solution of interior IHCP on analog apparatus, then as a rule the sampling method is used here [5], which, although it can be automated [6], is not the most efficient method of investigation, in our opinion, since it assumes the presence of a large quantity of controllable elements in the modeling medium. A somewhat different approach, when the initial mathematical model is converted to a form convenient for realization by the simplest means before the application of electrical modeling, should be considered more promising. The method of substitution, which we have utilized sufficiently successfully to solve direct [7] and exterior inverse [8] heat conductivity problems, can be used for such a conversion.

The application of substitution ordinarily results either in complete linearization of the initial mathematical model or in its simplification. Thus, the Kirchhoff substitution, which is used most often

$$\Theta = \int_0^T \lambda(T)\,dT \tag{1}$$

permits reduction of the nonlinear heat conduction equation

$$\nabla[\lambda(T)\nabla T] = c_V(T)\frac{\partial T}{\partial \tau} \tag{2}$$

to

$$\nabla^2\Theta = \frac{1}{a(\Theta)}\frac{\partial\Theta}{\partial\tau}, \tag{3}$$

and in the case of the stationary problem it permits reduction to the Laplace equation

$$\nabla^2\Theta = 0. \tag{4}$$

The initial conditions T = f(x, y, z, 0) and boundary conditions of the kind I-IV

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$$T_s = f(x, y, z, \tau), \quad -\lambda(T)\left(\frac{\partial T}{\partial n}\right)_s = f(x, y, z, \tau),$$

$$\alpha(x, y, z, \tau)(T_s - T_m) = -\lambda(T)\left(\frac{\partial T}{\partial n}\right)_s,$$

$$T_{1_s} = T_{2_s}, \quad -\lambda_1(T_1)\left(\frac{\partial T_1}{\partial n}\right)_{s_1} = -\lambda_2(T_2)\left(\frac{\partial T_2}{\partial n}\right)_{s_2}$$

are converted by using (1) into the expressions

$$\Theta = f(x, y, z, 0), \quad \Theta_s = f(x, y, z, \tau), \quad -\left(\frac{\partial \Theta}{\partial n}\right)_s = f(x, y, z, \tau),$$

$$\alpha(x, y, z, \tau)[T_s(\Theta) - T_m] = -\left(\frac{\partial \Theta}{\partial n}\right)_s, \tag{5}$$

$$T_{1_s}(\Theta_1) = T_{2_s}(\Theta_2), \quad -\left(\frac{\partial \Theta_1}{\partial n}\right)_{s_1} = -\left(\frac{\partial \Theta_2}{\partial n}\right)_{s_2}.$$

No constraints are imposed here on the domain of assignment of the heat conductivity operator.

Therefore, in the case of the stationary problem (or the nonstationary with $a(\Theta)$ = const), Eq. (2) is linearized and the mathematical model becomes either completely linear (under boundary conditions of the kinds I and II) or the nonlinearity goes over from the differential equation to the boundary conditions (nonlinear boundary conditions of the kinds III and IV).

The effect of applying the substitution (1) is reduced somewhat in solving the nonstatinary problem with $a = f(\Theta)$, since only the left side of (2) is linearized while the right side remains nonlinear. Nevertheless, even in this case the conversion of the mathematical model is meaningful, since the main modeling medium (the resistor network) need not be readjusted during the solution. The Schneider substitution results in an analogous result in the case of the linear dependence $\lambda(T)$.

The Goodman substitution

$$H = \int_0^T c_V(T)\,dT, \tag{6}$$

which reduces Eq. (2) to the form

$$\nabla[a(H)\nabla H] = \frac{\partial H}{\partial \tau}, \tag{7}$$

i.e., results in the right side of the equation becoming linear, is a no less interesting effect. Examples of the combined utilization of the substitutions (1) and (6) are known (see, e.g., [7]).

If the application of the mentioned substitutions was natural in the solution of the direct problems, then in the case of interior IHCP, when the dependences $\lambda(T)$, $c_V(T)$, and $a(T)$ are desired, the proposal to use these substitutions can be shown to be somewhat illogical. Meanwhile, if traditional representations about the modeling apparatus for the solution of heat conductivity problems are discarded, and they are replaced by self-adaptive apparatus, then the application of the substitutions mentioned turns out to be quite effective.

Let us consider examples of using the substitutions (1) and (6) for the identification of $\lambda(T)$, $c_V(T)$, and $a(T)$ in electrical models.

In all cases the results of thermometry at interior points of the bodies under investigation are taken as initial data. This can be separate temperature measurements as well as dependences characterizing the change in temperature with time.

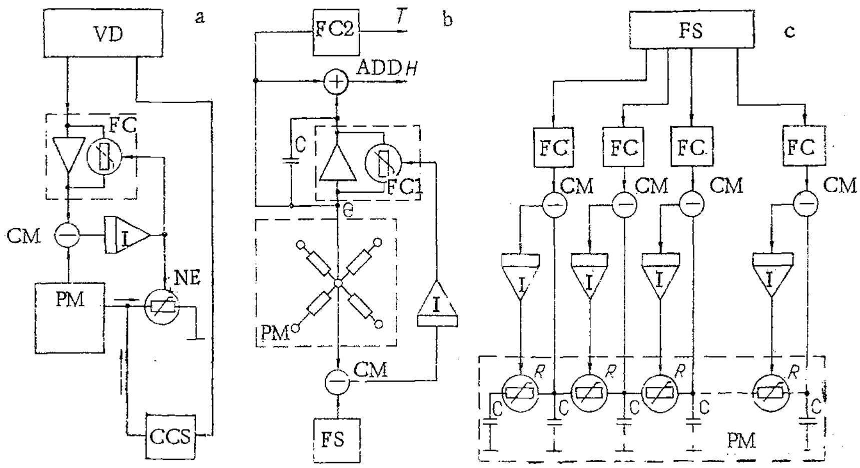

Fig. 1. Determination of the heat conduction coefficient (a), the volume specific heat (b), and the thermal diffusivity coefficient (c).

Determination of the Heat Conduction Coefficient by Using the Kirchhoff Substitution. In solving the stationary interior IHCP, after the conversion (1), Eq. (2) is transformed into the Laplace equation (4), which is modeled by a model of constant structure — a resistor network (R-network) — while the boundary condition of the kind III is the expression (5), which can be modeled by using a nonlinear element NE and a controllable current stabilizer CCS (Fig. 1a), as is done in solving direct problems [7] by the method of nonlinear resistors. The remaining circuit elements are to control the nonlinear element NE and the same element included in the feedback of the functional converter amplifier FC. A signal proportional to the given temperature at a certain point of the body under investigation goes from the voltage divider VD to the comparison module CM through the FC. The voltage from the corresponding nodal point of the passive model PM, whose role the above-mentioned R-network plays, is delivered to the second CM input. The mismatch signal from the CM output goes to the integrator I input, whose output signal is a control for the nonlinear elements. Regulation occurs until the mismatch signal becomes zero, i.e., until the voltage in the PM node will correspond to the temperature at this same point of the body being studied. Here self-adjustment of the NE and FC characteristics occurs. Since the NE and the nonlinear element of the FC are identical, while a current proportional to the first term in the left side of (5), i.e., the function $T(\Theta)$, is realized in the NE loop, then conversion of $\Theta(T)$ according to (1) occurs in the functional converter. The input and output signals of the functional converter, which are fixed during the experiment, afford the possibility of constructing the dependence $\lambda(T)$.

Determination of the Specific Volume Capacity by Using the Kirchhoff and Goodman Substitutions. A nonstationary interior IHCP is solved, where the substitution (1) is applied to convert the left side of (2), and the substitution (6) is applied to convert its right side. Consequently Eq. (2) becomes

$$\nabla^2\Theta = \frac{\partial H}{\partial \tau}, \tag{8}$$

and the left side of (8) can be modeled by a passive model (R-network), as in the preceding example, and a nonlinear capacitance similar to the apparatus described in [7] can be used for modeling the right side. This nonlinear capacitance includes the constant capacitor C (Fig. 1b), to which the functional converter FC1 with the controllable resistor in the feedback is included in parallel between the passive model PM and the adder ADD. A servo system analogous to the servo system of the preceding apparatus (comparison module CM, included between the PM and the output of the functional shaper FS in which the signal proportional to the temperature change at a certain point of the body being modeled is shaped, and the integrator I) controls the FC characteristic. A certain dependence

$$F = \Theta - H = f(\Theta) \tag{9}$$

is realized in the functional converter, and a current proportional to $\partial/\partial\tau\ (\Theta - F)$, flows through the capacitance C, and taking (9) into account, it will also be proportional to $\partial H/\partial\tau$. Simultaneously, the voltage on the nodal point PM proportional to the function Θ goes to the adder input and functional converter FC2 input, whereupon a voltage proportional to the function H is shaped at the ADD output, and a signal proportional to the temperature is at the output of FC2, realizing the known dependence $T(\Theta)$. Therefore, the apparatus considered [9] permits the construction of the function H(T), from which the dependence $c_V(T)$ is determined, by recording both dependences.

Determination of the Thermal Diffusivity Coefficient by Using the Goodman Substitution. The nonstationary internal IHCP is solved, i.e., Eq. (2) after the conversion (6) takes the form (7). The right side of (7) can be realized by using constant capacitors, and the left side can be realized by using controllable resistive elements, i.e., (7) can be modelled by a capacitance–resistor network (RC-network) if its resistors are made controllable [10]. To do this, the nodal point of the passive model (RC-network) is connected to the comparison module (Fig. 1c), whose second input is connected through the functional converter FC to the functional shaper FS, in which a signal is formed proportional to the temperature change at the corresponding point of the body being modeled. Conversion according to (6) is accomplished in the functional converter. The mismatch signal from the comparison module output goes to the input of the integrator I, whose output signal controls the resistive element R connected between the nodal points of the passive model PM. Regulation occurs until the mismatch signals become zero, i.e., changes of the voltage at the passive model nodes correspond to temperature changes at corresponding points of the body under investigation. The resistance of the controllable resistive element, measured during regulation, permits a judgment about the dependence $a(T)$ – the thermal diffusivity coefficient and the measurable resistance are related in inverse proportion to the dependence.

It should be noted that the proposed means for solving interior IHCP differ favorably from other methods of identifying the thermophysical properties of materials. Firstly, they permit utilization of results of ordinary thermometry without execution of special experiments; information obtained directly on full-scale objects and thermal models turns out to be sufficient. Secondly, the geometry of the objects under investigation can be arbitrary, while special experimental specimens assuring a uniform temperature field are ordinarily required in other methods. Thirdly, in comparison to the method of sampling, the proposed method permits more economical execution of the modeling, since the time expenditures turn out to be so very low (in the last two examples) and the modeling facilities used contain so few controllable elements, especially in the first case, when the interior IHCP actually reduces to the exterior problem.

As regards the regularity of the inverse problem solutions obtained on analog apparatus, certain considerations about this were expressed in [11] (as a rule, the comparatively low accuracy of electrical models does not permit the solution to drop into the instability zone). If the approach is rigorous, then application of analog facilities does not deprive the examiner of the necessity to apply special measures that regularize the solution or determine the boundaries in which it will remain regular. These can be conditions for halting the iteration process, and constraints on the feedback gain coefficient (which exist in apparatus for solving inverse problems), on the site locating the points yielding the initial information, on the step of the finite-difference approximation, etc. In solving interior IHCP it is sometimes necessary to have the presence of reference values of the desired thermophysical characteristics at definite temperatures (in the case of boundary conditions of the kind I) in order to obtain a unique solution. The accuracy of the solution depends on the quantity of these reference values, but since the nature of the dependence is found during the solution, in principle one such point is sufficient to assure uniqueness. The presence of reference values is not certain in problems with boundary conditions of the second, third, and fourth kinds, since there is complete definiteness with respect to the flux of the thermal energy on the boundaries of the object under investigation.

In conclusion, let us present results of identifying the thermophysical properties of diamond polycrystals ASB, performed by S. F. Lushpenko by using the apparatus described on the basis of a thermophysical experiment performed by V. V. Rusanov. The mean specific heat of polycrystalline diamond and dependences of its heat conduction coefficient on the temperature were determined. The magnitude of the mean specific heat of the polycrystal $c_p = 1330$ kJ/(m^3 · deg K) turned out to be of the same order as the specific heat of a synthetic diamond single crystal $c_s = 1760$ kJ/(m^3 · deg K). The nature of the obtained

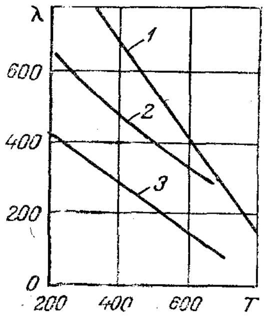

Fig. 2. Results of identifying the dependence λ(T) for polycrystalline diamond.

dependence $\lambda_p = f(T)$ on the temperature (curve 3 in Fig. 2) is in good agreement with the known data for natural diamond (curve 1) and the single crystal (curve 2); however, the general level of λ_p is lower, which can be explained by the presence of gas pores in the polycrystal, and of a definite percentage of metal catalyst, its compounds, and other impurities whose heat conduction is considerably below the heat conduction of natural diamond and the synthetic single crystal.

NOTATION

T, temperature, °K; λ, thermal conductivity, W/(m · deg); c_V, specific volume heat capacity, J/(kg · deg); a, thermal diffusivity, m^2/sec; τ, time, h, sec; α, heat-transfer coefficient, W/(m^2 · deg). Subscripts: s, surface; m, medium.

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